Key Components in Methodology Formulation

Entec

R&D Technical Report W10

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GLOSSARY

Note:
Many abbreviations and acronyms used in this project are highly specific and have no recognised meaning elsewhere.

ω (d^{-1})	Aquifer Parameter 'omega' derived by the Water Resources Board
α (d^{-1})	Aquifer Parameter 'alpha' derived in this project
ABF	Average Baseflow (sometimes expressed as a discharge 'Ml/d' or a yield 'l/s/km^2')
ADF	Average Daily Flow, expressed as a discharge (i.e. Ml/d)
ADIST (km)	Composite Distance of Abstractions from the River
AQCONF	Aquifer Configuration
AQGEOL	Aquifer Geology
AREAR (km^2)	Area of Aquifer Receiving Recharge
AREAW (km^2)	Area of Whole Aquifer
ARQ (Ml/a)	Total Aquifer Recharge Quantity
BHS	British Hydrological Society
BFS	Baseflow Significance of Aquifer unit
C (m^2/d)	River Bed Conductance, used in MODFLOW analysis
D_S	Safe Development Limit of an Aquifer, as a proportion of Recharge input
DSG	Drift Sand and Gravel
D_T	Aquifer Development Threshold, as a proportion of Average Baseflow (or Recharge input)
E_A (Ml)	Total Annual Effluent Return to the River
EA	Environment Agency
EAL (km)	Effective Aquifer Length (derived using a simple formula based on catchment area and river length)
EC	Environmental Criticality of River

E_i (Ml)	Individual Effluent Return Quantity
E_S (Ml)	Total Summer Effluent Return to the River
E_W (Ml)	Total Winter Effluent Return to the River
F_{SD}	Seasonality-Distance Factor
F_T	Transmissivity Factor from Aquifer to River
h (m)	Groundwater head
ho (m)	Constant head at rivers (fixed head in MODFLOW analysis at river cell)
HoF	Hands off Flow - flow in a river below which licences incorporating a related cessation clause must cease abstraction
HS	Hydrological Sensitivity of River
ICL	Interfluvial Chalk/Limestone plus miscellaneous hardrock aquifers
K (m/d)	Aquifer Hydraulic Conductivity
K_b (m/d)	River Bed Hydraulic Conductivity
KCL	Karstic Chalk/Limestone
L (m)	Aquifer length, used in MODFLOW analysis
L_A (Ml)	Total Licensed Annual Quantity
L_i (Ml)	Individual Licence Quantity
L_S (Ml)	Total Licensed Summer Quantity
L_W (Ml)	Total Licensed Winter Quantity
MORECS	Meteorological Office Rainfall and Evaporation Calculation System
NRA	National Rivers Authority
OSST	Other Sandstones (Cretaceous, Carboniferous etc)
PABF	Proportion of Average Baseflow (or Recharge) necessary to protect baseflow requirements
PAR	Proportion of Average Recharge necessary to protect other environmental needs
Q_{IW} (Ml/a)	Intermediate Weighted Assessment of Development

Qn_x	Naturalised river flow percentile exceedence values with x defining the percentile exceedence value
Q_S (Ml/a)	Safe *Yield* of an Aquifer, also referred to as the Groundwater Resource Reliable Yield
Q_{SW} (Ml/a)	Simple Weighted Assessment of Development
quasi-steady-state	MODFLOW simulation results using a cyclic input of Average Monthly Recharge data
R&D	Research and Development
RECH (mm/a)	Annual Average Recharge
RIVLEN (km)	River Length
RS_A (Ml)	Total Annual River Support Quantity
RS_i (Ml)	Individual River Support Quantity
RS_S (Ml)	Total Summer River Support Quantity
RS_W (Ml)	Total Winter River Support Quantity
S	Aquifer Storativity (sometimes S_y is substituted)
SEAS	Seasonality Factor of Abstractions
SSST	Sherwood Sandstone
S_y	Specific Yield
T (m^2/d)	Aquifer Transmissivity
T' (m^2/d)	"Effective" Transmissivity from Aquifer to River
transient	MODFLOW simulation results using an actual historic monthly recharge dataset as input
VCL	Typical Valley Chalk/Limestone
WRB	Water Resources Board
x	i) 'aquifer length' (m) applicable to the Dupuit equation ii) 'percentile exceedence' applicable to naturalised flow percentile exceedence values

EXECUTIVE SUMMARY

Although this R&D Technical Report is compiled as a 'stand alone' document it is intended to complement the User Manual which describes the formulation and application of the Groundwater Resource Reliable Yield (*yield*) Methodology.

This document provides details of the groundwater modelling and analytical work which underpins the formulation of the basic 'type curve' approach and many of the factors used in the resulting Methodology.

The details in this document generally relate to an idealised aquifer configuration and the main focus demonstrates how the baseflow regime can be readily defined by a relatively simple consideration of aquifer recharge and typical aquifer characteristics. In most cases these data will be available, albeit with some uncertainties, enabling assessments to be conducted.

This document does not describe the environmental constraints and allocations which also limit the resulting *yield* quantity. These are defined in the User Manual.

Keywords

Groundwater, Methods, Resources, Yield

1. INTRODUCTION

This R&D Technical Report includes an account of the scientific work carried out that forms the basis for some of the procedures and factors used in the Groundwater Resource Reliable Yield (*yield*) Methodology as described in the R&D Technical Report W9 - User Manual (Papaioannou and Erskine, 1996a). Therefore, whilst this R&D Technical Report is written as a 'stand alone' document, its total meaning and application can only be realised if read in conjunction with the User Manual.

The details described here only apply when the baseflow from the aquifer to surface water is considered significant as described in Section 2.1 of the User Manual. This account only elucidates procedures and factors used in the Methodology (described in the User Manual) which are derived from computer modelling and related analytical assessment. Most of these form the basis for deriving the 'alpha' parameter and applying this to a 'type curve' as a means of establishing D_T, the seasonal minimum baseflow expressed as a fraction of the average baseflow, in an idealised and non developed (zero groundwater abstraction) aquifer.

Other factors and procedures used in the Methodology to determine *yield* (Q_S) are not considered here but are detailed in the User Manual. These include parameters such as:

- PABF the proportion of average baseflow allocated to protect minimum baseflow regime.

- PAF the proportion of average baseflow or recharge to protect 'other' groundwater related environmental needs.

To date, no comparable consideration or evaluation has been undertaken to deal with the situation when the baseflow from an aquifer is not considered significant. The procedures described in the User Manual under these circumstances has not been subject to any scientific evaluation.

A detailed Introduction and Background to the project is given in Section 1 of the R&D Project Record (Papaioannou, 1996).

The second section in this Note is a description of the modelling runs carried out and a summary of the lessons learnt during the experimentation. The purpose of this work was to determine the minimum baseflows to rivers resulting from seasonal fluctuations in recharge to an idealised aquifer.

The third section demonstrates the existence of a key 'aquifer parameter' which can be derived from the idealised assumptions used in the modelling. This parameter enables output from the model to be described mathematically. Therefore, it is possible to distil the findings from the modelling work down to a single equation and 'type curve'.

In the fourth section work is described that tests the theories advanced in sections two and three. Real data is analysed in order to put the 'type curve' on a sound empirical basis. Hydrological statistics have been used to verify the equations and show that they generally represent real conditions in gauged catchments satisfactorily and therefore, the method can be applied to both highly investigated/monitored areas as well as ungauged catchments. Further case examples are given in Section 3.2 of the User Manual.

2. MODELLING SET-UP AND EXPERIMENTATION

2.1 Set-up

In order to investigate the behaviour of an idealised aquifer under variable recharge conditions, a simple numerical model of the aquifer was developed. This model used the software MODFLOW which employs the method of finite differences to approximate the Darcy equation of groundwater flow.

The model consists of a 1-dimensional unconfined aquifer in section with the river forming a boundary at one end (represented by a fixed head node) and a groundwater divide at the other end (represented by a no-flow node) as shown in Figure 2.1. The equation of groundwater flow used is:

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)=S\frac{\partial h}{\partial t}-q$$

The initial variables considered in the modelling experimentation were the head at the river boundary (ho), the hydraulic conductivity (K) of the aquifer, the storage (S) of the aquifer and the length (L) of the aquifer.

A large number of model runs not involving any abstraction simulations were carried out which can be divided into the following categories:

- Steady State runs. Used to verify the model with the analytical solution $h^2 = qx\,(2L - x)/K + ho^2$ where x is distance from the river.

- 'Quasi-Steady-State' Runs. Transient runs where the recharge was set to an annual cycle. The average monthly figures (derived over the period 1961-1990) for MORECS square 109 were used for the annual recharge. After a few years the flow to the river also becomes cyclical. The minimum figure in the river flow cycle is noted as a percentage of the average flow. This ratio is defined as D_T or the ratio of Minimum Base Flow (MBF) to Average Base Flow (ABF). *In order to standardise output from the different model configurations and enable meaningful comparisons, much of the output shown in hydrograph form uses yield (l/s/km^2) rather than flow (l/s).*

- Transient Runs. Transient Runs using historic monthly data for MORECS square 109 (see Figure 2.2). The runs were conducted for the 30 year period (1960-1990) using monthly time steps and the initial condition was the 'Quasi-steady-state' solution. The output of these runs was noted in terms of the minimum annual baseflow yields to the river with each annual minimum value represented on a Gumbel Probability Plot.

Output from the above model runs were used to form the basis for defining a key 'aquifer parameter' which could be derived mathematically from the initial variables described above.

Further 'Quasi-Steady-State' modelling experimentation was then conducted to explore the effect of:

- Abstraction Scenarios. For selected runs, abstractions were included in the model. The effects of varying abstraction season and location were also briefly investigated to establish a factor (F_{SD}) which could be applied in the Methodology.

- Variable River Bed Conductance Scenarios. For selected model runs and aquifer parameter values the effect of reducing river bed conductance was briefly explored. From this it was possible to show how the 'effective transmissivity' (T') from the aquifer to the river is modified by different combinations of aquifer transmissivity and river bed properties leading to formulation and application of a transmissivity factor (F_T).

- Spatially Variable Hydraulic Conductivity (Spatially Variable K). For selected model runs the effect of spatially variable hydraulic conductivity (typical of Chalk aquifers) was very briefly explored. There is a strong inference that applying the 'weighted' mean (composite) K across the whole aquifer model gives a reasonable approximation to the output from spatially variable K model runs. Therefore, this simple approximation is recommended in the Methodology.

2.2 Results

2.2.1 General Findings

The results of the steady-state runs were in accordance with the analytical solutions.

The results of the 'quasi-steady-state' runs are given in Table 2.1. A selection of the initial runs are also illustrated in Figure 2.3 and 2.4. The model was run for ten years and the baseflow yields to the river in the last year (by which time the annual pattern is repeating identically) are presented. Clearly the minimum yields becomes smaller as:

- K increases
- S decreases
- the fixed head (ho) increases
- the aquifer length (L) decreases.

The minimum yield (or flow) expressed as a proportion of average baseflow yield is a measure of the sensitivity of the baseflow as related to the aquifer parameters.

Briefly, minimum baseflow yields are shown to:

- reduce in model runs with abstraction simulations. Minimum yields are most dominantly affected by abstraction quantity but are also sensitive to abstraction season and location with the summer season and locations right next to the river proving the most sensitive.

- increase in model runs with reduced river bed conductance (usually resulting from reduced river bed hydraulic conductivity) which will cause 'effective transmissivity' (T') to reduce when compared with aquifer transmissivity T.

The relationships between all the parameters described above and the minimum baseflow yield is further developed in Section 3.

The results of various runs (for ho = 1 m and L = 1000 m) with transient data are shown in Figures 2.5, 2.6 and 2.7. Probability Plots using the Annual Minima are shown in Figures 2.8, 2.9 and 2.10. A complete list of runs carried out is given in Table 2.2.

Table 2.1 List of MODFLOW runs carried out

Model Run Number	Recharge q md^{-1}	Hydraulic Conductivity K md^{-1}	Storativity S -	Aquifer Length L m	Fixed Head h_o m	Equivalent Head (h) = $\sqrt{(h_o^2+qL^2/K)}$ m	Alpha $T/L^2.S$ d^{-1}	MBF ABF D_T -
QT1	3.76E-04	1	0.01	1000	1	19.4165	0.0019	0.653
QT2	3.76E-04	5	0.01	1000	1	8.7293	0.0044	0.399
QT3	3.76E-04	20	0.01	1000	1	4.4497	0.0089	0.192
QT4	3.76E-04	50	0.01	1000	1	2.9189	0.0146	0.091
QT5	3.76E-04	5	0.03	1000	1	8.7293	0.0015	0.687
QT6	3.76E-04	10	0.03	1000	1	6.2129	0.0021	0.602
QT7	3.76E-04	20	0.03	1000	1	4.4497	0.0030	0.501
QT8	3.76E-04	50	0.03	1000	1	2.9189	0.0049	0.350
QT9	3.76E-04	5	0.05	1000	1	8.7293	0.0009	0.774
QT10	3.76E-04	20	0.05	1000	1	4.4497	0.0018	0.633
QT11	3.76E-04	50	0.05	1000	1	2.9189	0.0029	0.497
QT12	3.76E-04	1	0.1	1000	1	19.4165	0.0002	0.947
QT13	3.76E-04	3	0.1	1000	1	11.2398	0.0003	0.889
QT14	3.76E-04	10	0.15	1000	1	6.2129	0.0004	0.858
QT15	3.76E-04	1	0.25	1000	1	19.4165	0.0001	0.984
QT16	3.76E-04	5	0.25	1000	1	8.7293	0.0002	0.932
QT17	3.76E-04	50	0.25	1000	1	2.9189	0.0006	0.808
QT2B	3.76E-04	5	0.01	1000	15	17.3263	0.0087	0.126
QT9B	3.76E-04	5	0.05	1000	15	17.3263	0.0017	0.594
QT16B	3.76E-04	5	0.25	1000	15	17.3263	0.0003	0.817
QT2C	3.76E-04	5	0.01	1000	50	50.7464	0.0254	0.010
QT9C	3.76E-04	5	0.05	1000	50	50.7464	0.0051	0.262
QT16C	3.76E-04	5	0.25	1000	50	50.7464	0.0010	0.637
QT4B	3.76E-04	50	0.01	1000	15	15.2486	0.0762	0.003
QT11B	3.76E-04	50	0.05	1000	15	15.2486	0.0152	0.021
QT17B	3.76E-04	50	0.25	1000	15	15.2486	0.0030	0.429
QT4C	3.76E-04	50	0.01	1000	50	50.0751	0.2504	0.000
QT11C	3.76E-04	50	0.05	1000	50	50.0751	0.0501	0.003
QT17C	3.76E-04	50	0.25	1000	50	50.0751	0.0100	0.083

Table 2.1 List of MODFLOW runs carried out (continued)

Model Run Number	Recharge q md^{-1}	Hydraulic Conductivity K md^{-1}	Storativity S -	Aquifer Length L m	Fixed Head h_o m	Equivalent Head (h) = $\sqrt{(h_o^2+qL^2/K)}$ m	Alpha $T/L^2.S$ d^{-1}	MBF ABF D_I -
K1S1L1	3.76E-04	5	0.01	300	5	5.6363	0.0313	0.009
K1S1L2	3.76E-04	5	0.01	500	5	6.6182	0.0132	0.056
K1S1L3	3.76E-04	5	0.01	1000	5	10.0100	0.0050	0.324
K1S1L4	3.76E-04	5	0.01	3000	5	26.4915	0.0015	0.675
K1S1L5	3.76E-04	5	0.01	5000	5	43.6463	0.0009	0.772
K1S2L1	3.76E-04	5	0.05	300	5	5.6363	0.0063	0.204
K1S2L2	3.76E-04	5	0.05	500	5	6.6182	0.0026	0.490
K1S2L3	3.76E-04	5	0.05	1000	5	10.0100	0.0010	0.722
K1S2L4	3.76E-04	5	0.05	3000	5	26.4915	0.0003	0.884
K1S2L5	3.76E-04	5	0.05	5000	5	43.6463	0.0002	0.927
K1S3L1	3.76E-04	5	0.25	300	5	5.6363	0.0013	0.658
K1S3L2	3.76E-04	5	0.25	500	5	6.6182	0.0005	0.789
K1S3L3	3.76E-04	5	0.25	1000	5	10.0100	0.0002	0.893
K1S3L4	3.76E-04	5	0.25	3000	5	26.4915	0.0001	0.970
K1S3L5	3.76E-04	5	0.25	5000	5	43.6463	0.0000	0.988
K2S1L1	3.76E-04	50	0.01	300	5	5.0672	0.2815	0.012
K2S1L2	3.76E-04	50	0.01	500	5	5.1846	0.1037	0.006
K2S1L3	3.76E-04	50	0.01	1000	5	5.7026	0.0285	0.009
K2S1L4	3.76E-04	50	0.01	3000	5	9.6270	0.0053	0.299
K2S1L5	3.76E-04	50	0.01	5000	5	14.5945	0.0029	0.495
K2S2L1	3.76E-04	50	0.05	300	5	5.0672	0.0563	0.018
K2S2L2	3.76E-04	50	0.05	500	5	5.1846	0.0207	0.006
K2S2L3	3.76E-04	50	0.05	1000	5	5.7026	0.0057	0.235
K2S2L4	3.76E-04	50	0.05	3000	5	9.6270	0.0011	0.710
K2S2L5	3.76E-04	50	0.05	5000	5	14.5945	0.0006	0.805
K2S3L1	3.76E-04	50	0.25	300	5	5.0672	0.0113	0.060
K2S3L2	3.76E-04	50	0.25	500	5	5.1846	0.0041	0.336
K2S3L3	3.76E-04	50	0.25	1000	5	5.7026	0.0011	0.676
K2S3L4	3.76E-04	50	0.25	3000	5	9.6270	0.0002	0.887
K2S3L5	3.76E-04	50	0.25	5000	5	14.5945	0.0001	0.934

Table 2.1 List of MODFLOW runs carried out (continued)

Model Run Number	Recharge q md^{-1}	Hydraulic Conductivity K md^{-1}	Storativity S -	Aquifer Length L m	Fixed Head h_o m	Equivalent Head (h) = $\sqrt{(h_o^2+qL^2/K)}$ m	Alpha $T/L^2.S$ d^{-1}	MBF ABF D_I -
ABR1	3.76E-04	1	0.03	1000	5	20.0250	0.0007	0.800
ABR2	3.76E-04	5	0.03	1000	5	10.0100	0.0017	0.634
ABR3	3.76E-04	10	0.03	1000	5	7.9120	0.0026	0.508
ABR4	3.76E-04	20	0.03	1000	5	6.6182	0.0044	0.340
ABR5	3.76E-04	50	0.03	1000	5	5.7026	0.0095	0.110
SPK1	3.76E-04	18.5	0.03	1000	5	6.7323	0.0042	0.424
SPK2	3.76E-04	3.7	0.03	1000	5	11.2526	0.0014	0.611
VRBC1	3.76E-04	10	0.03	1000	5	7.9120	0.0026	0.341
VRBC2	3.76E-04	1	0.03	1000	5	20.0250	0.0007	0.356
VRBC3	3.76E-04	0.1	0.03	1000	5	61.5224	0.0002	0.468
VRBC4	3.76E-04	10	0.03	1000	5	7.9120	0.0026	0.113
VRBC5	3.76E-04	1	0.03	1000	5	20.0250	0.0007	0.128
VRBC6	3.76E-04	0.1	0.03	1000	5	61.5224	0.0002	0.338
VRBC7	3.76E-04	1	0.03	1000	5	20.0250	0.0007	0.514
VRBC8	3.76E-04	0.1	0.03	1000	5	61.5224	0.0002	0.578

Notes:

QT1 etc	Transient Flow runs - Basic Model Runs based on Steady State analysis.
K1S1L1 etc	Variations and combinations of Aquifer K, S and length.
ABR1 etc	Additional Baseline runs with S = 0.03 and L = 1000m.
SPK1 etc	Spatially Variable K analysis runs.
VRBC1 etc	Variable River Bed Conductance runs.

Table 2.2

Summary of Transient MODFLOW Runs Carried Out Using Historical Data

Run Number	Aquifer Parameters				Minimum Flow		
	S	K m/d	ho m	L m	Median % ABF	20 year Return period % ABF	Quasi-Steady Solution % ABF
HIST2	0.01	5	1	1 000	26	9	40
HIST4	0.01	50	1	1 000	2	0	9
HIST9	0.05	5	1	1 000	71	42	77
HIST11	0.05	50	1	1 000	34	13	50
HIST16	0.25	5	1	1 000	90	73	93
HIST17	0.25	50	1	1 000	77	48	81

An important observation drawn from these runs is that the variability of the real annual recharge causes minimum baseflow yields to generally be lower than those predicted from the quasi-steady-state model. Even the 'average' (median) minimum baseflow yield is always less than the 'quasi-steady-state' solution. This probably results from the inherent variability in each individual year and it is believed that taking a 'quasi-steady-state' solution from average monthly recharge figures (derived using monthly data for MORECS grid square 109 over the period 1961 to 1990) gives a recharge input during critical recession periods which is relatively moderated in severity (i.e. truncated in both spring and autumn by averaging) and which is not indicative or typical of cyclic recession characteristics on an individual annual basis.

2.2.2 The Effects of Annual and Seasonal Abstractions

The 'quasi-steady-state' runs involving abstractions and seasonality are summarised in Table 2.3. The size of the abstraction has been set, using assumptions and output from the runs outlined in Table 2.1, as the amount of water that would reduce the residual minimum baseflow yields to 20% of average (without abstraction) if abstracted continuously at a location next to the river. The percentages given for the various runs are the revised residual minimum baseflow yields.

Table 2.3

Effects on Residual Minimum Baseflow Yield caused by Abstraction Rate

S	K	Abstraction Rate	Residual Minimum Baseflow Yield (% ABF)				
			Abstraction close to River			Abstraction spread out	
	m/d	% ABF	All year (continuous)	Winter only	Summer only	All year (continuous)	Summer only
0.01	5	17.6	20.2	37.5	20.4	22.4	23.6
0.05	50	26.3	20.3	35.8	20.6	22.7	25.5
0.05	5	50.9	20.2	33.3	22.1	22.9	34.9
0.25	50	53.8	20.4	33.0	22.3	22.4	36.2
0.25	5	64.8	21.3	41.3	27.5	31.8	47.6

All the above runs were conducted with ho = 1 m and L = 1000 m and by default with $F_T = 1.0$ *(i.e. river bed hydraulic conductivity is equivalent to aquifer hydraulic conductivity).*

When the 'residual' percentage is larger than 20% this indicates extra resource being available for development. The seasonal columns ('winter only' etc) represent abstractions at the same rate but only for half the year and therefore the total abstraction quantity is half the equivalent 'all year' value.

The conclusions to be drawn are:

i) if abstraction is located further away from the river then some 'extra' water becomes available for abstraction development

ii) if abstraction is in winter only, considerably more resource can usually be exploited

iii) if abstraction is in summer only, the resource is usually and almost as depleted as if it were abstracted all the year round.

This work has emphasised the need to consider where abstraction takes place in the catchment and the particular magnitude of summer abstractions. Therefore, these limited results have been used to derive a factor (F_{SD}) which is applied in the Methodology as highlighted in the User Manual (see Section 2.2.4 and Table 2.6 therein).

2.2.3 The Effects of Variable Aquifer K and River Bed Conductance

A small series of 'quasi-steady-state' model runs were conducted to explore the effects of:

- Spatially Variable Aquifer Hydraulic Conductivity
- Variable River Bed Conductance.

The baseline runs used for comparison with the above are all outlined in Table 2.1 and include fixed hydraulic or geometric parameters of;

- ho = 5 m;
- L = 1000 m;
- S = 0.03;
- river bed hydraulic conductivity effectively equal to aquifer hydraulic conductivity;

and were modelled using varied aquifer hydraulic conductivities of 1, 5, 10, 20 and 50 m/d. The results from the baseline runs, with output expressed as baseflow yield (l/s/km^2) are shown in Figure 2.11.

The baseline and exploratory model runs outlined above are summarised in Table 2.4 and the results are summarised below.

i) Spatially Variable Aquifer Hydraulic Conductivity

The 1000 m length of aquifer was divided into two zones of contrasting hydraulic conductivity with:

- K_1 going from the river cell boundary (at zero) to 300 m distant.
- K_2 going from 300 m distant to the model boundary (at 1000 m).

Two model runs were conducted with:

- K_1 = 50 m/d and K_2 = 5 m/d
- K_2 = 10 m/d and K_2 = 1 m/d.

Table 2.4 Exploring the effects of variable aquifer hydraulic conductivity and river bed Conductance

Year 10 (month-end) MODFLOW Baseflow Yields at the River Cell (in l/sec/km^2)

	Additional Baseline Runs					Spatially Variable K (m/day)		Variable River Bed Conductance							
								K = 20m/day			K = 50m/day			K = 10m/day	
	K = 1m/d	K = 5m/d	K = 10m/d	K = 20m/d	K = 50m/d	K= 50 and 5 K' = 18.5	K= 10 and 1 K' = 3.7	K_b = 10 C=5000m^2/	K_b = 1 C=500m^2/d	K_b = 0.1 C=50m^2/d	K_b = 10 C=5000m^2/	K_b = 1 C=500m^2/d	K_b = 0.1 C=50m^2/d	K_b = 1 C=500m^2/d	K_b = 0.1 C=50m^2/d
Month \ Run:	ABR1	ABR2	ABR3	ABR4	ABR5	SPK1	SPK2	VRBC1	VRBC2	VRBC3	VRBC4	VRBC5	VRBC6	VRBC7	VRBC8
Dec.09	4.4273	4.5222	4.6161	4.9500	6.3854	5.6514	5.1271	4.9439	4.8370	4.1544	6.3567	6.0449	4.2775	4.5688	4.2122
Jan.10	5.3391	6.1848	6.8407	7.9975	10.6060	8.4219	7.0311	7.9873	7.8013	6.3683	10.5688	10.1454	7.0760	6.7528	5.9926
Feb.10	5.4734	6.4010	7.1242	8.3198	10.3052	7.9528	6.8910	8.3138	8.1946	7.0861	10.2906	10.1096	7.9804	7.0675	6.5080
Mar.10	5.4789	6.3778	7.0720	8.0786	9.1228	7.3912	6.5063	8.0753	8.0110	7.2781	9.1226	9.1078	8.0792	7.0369	6.6551
Apr.10	5.1442	5.7315	6.1683	6.6635	6.5703	5.8556	5.4909	6.6644	6.6786	6.5716	6.5852	6.7365	7.0579	6.1690	6.1029
May.10	4.5509	4.6223	4.6472	4.4758	3.3017	3.7708	4.0884	4.4806	4.5691	5.1259	3.3271	3.6037	5.1550	4.6846	4.9656
Jun.10	4.1762	3.9377	3.7098	3.1723	1.6736	2.8506	3.3969	3.1778	3.2767	4.0253	1.6954	1.9396	3.7130	3.7535	4.1257
Jul.10	3.9061	3.4578	3.0639	2.3378	0.8899	2.3497	3.0211	2.3428	2.4344	3.2020	0.9057	1.0837	2.6839	3.1064	3.4917
Aug.10	3.6831	3.0713	2.5627	1.7490	0.4801	1.9689	2.7561	1.7537	1.8359	2.5736	0.4904	0.6141	1.9569	2.6034	2.9867
Sep.10	3.5426	2.8398	2.2821	1.4796	0.4843	1.8461	2.6590	1.4832	1.5495	2.1900	0.4900	0.5583	1.5609	2.3174	2.6630
Oct.10	3.4824	2.7579	2.2109	1.5039	0.8692	1.9661	2.7258	1.5064	1.5506	2.0368	0.8696	0.8802	1.4698	2.2373	2.5139
Nov.10	3.7635	3.2913	2.9543	2.6160	2.7981	3.2645	3.5398	2.6146	2.5964	2.6266	2.7860	2.6532	2.2894	2.9509	2.9810
Dec.10	4.4340	4.5222	4.6161	4.9500	6.3854	5.6514	5.1265	4.9439	4.8370	4.1544	6.3567	6.0449	4.2775	4.5688	4.2122
					Comparable Baseline Run	ABR4	ABR2	ABR4	ABR4	ABR3	ABR5	ABR5	ABR4	ABR3	ABR2/3
					Suggested Factor	N/A	N/A	1	0.95	0.6	1	0.93	0.4	0.99	0.75

All data based on model parmeters as follows: $S = 0.03$, $L = 1000m$, $h_o = 5m$

C = River Bed Conductance

K = Aquifer Hydraulic Conductivity

K_b = River Bed Hydraulic Conductivity

K' = Area Weighted Average Value

When modelled output is compared with that from the baseline runs (see Figure 2.12 and Table 2.4), and concentration is given to the minimum modelled yield, it can be seen that the most similar matches are achieved with K = 20 m/d and K = 5 m/d respectively. Although there is cyclic distortion in the resultant yield hydrographs, when making comparisons, it is concluded that the weighted mean of the variable aquifer hydraulic conductivity (composite K) should give an adequate approximation for Methodology purposes. In these examples composite K equates to 18.5 m/d and 3.7 m/d respectively.

ii) Variable River Bed Conductance

Variable River Bed Conductance has been modelled by effectively assuming a 1 m width of river bed material next to the river cell with a range of hydraulic conductivity values expressed as K_b. With the baseline model configuration this gives river bed conductance (C) expressed as:

$$C = \frac{K_b * x * ho}{b}$$

where b = river bed thickness (equal to 1 m in this instance)
x = width of modelled river cell (equal to 100 m in this instance).

Therefore, river bed conductance reduces to:

$$C = 500 * K_b$$

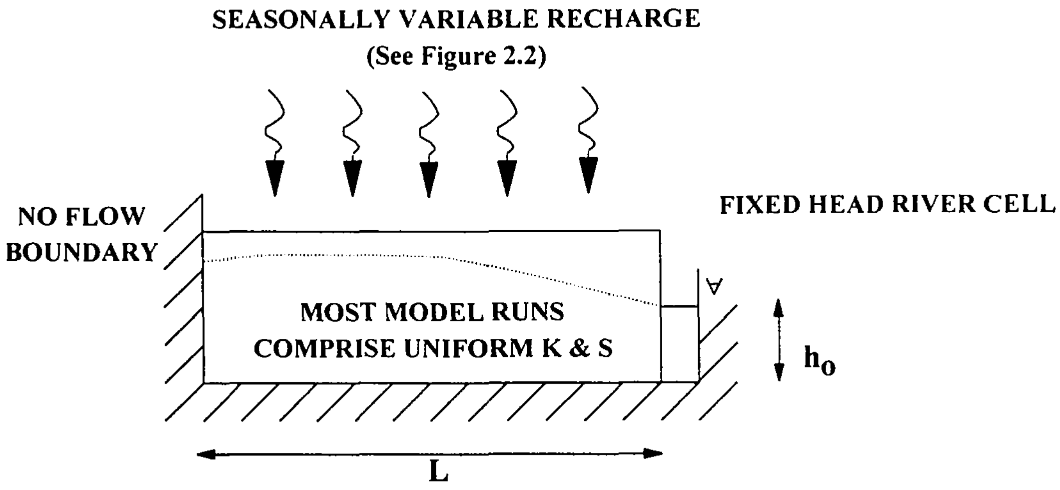
The K_b ranges explored and resultant C values considered were:

- $K_b = 0.1$ m/d (equivalent to silt) giving $C = 50\ m^2/d$
- $K_b = 1.0$ m/d (equivalent to silty sand) giving $C = 500\ m^2/d$
- $K_b = 10.0$ m/d (equivalent to medium sand) giving $C = 5000\ m^2/d$.

and where K_b is less than aquifer K, model runs have been conducted with aquifer K equal to 10 m/d, 20 m/d and 50 m/d (see Table 2.4). All runs have been conducted using 'quasi-steady-state' analysis and the comparisons between modelled yield output for variable river bed conductance and baseline values are shown in Figures 2.13, 2.14 and 2.15 respectively. In addition, Table 2.4 also highlights, for each run with aquifer K > river bed K_b, which baseline run gives the most similar resultant yield.

In addition, Table 2.4 also includes a suggested factor, which for the model configuration including a river bed conductance term, expresses the reduction necessary to the aquifer K value, when used in the model with no river bed conductance component (as in the baseline runs), in order to give an equivalent modelled output.

In the Methodology, as described in the User Manual (Papaioannou and Erskine, 1996a) Section 2.2.2 and Table 2.3, these 'same' factors are expanded to reduce aquifer transmissivity (T) to 'effective' transmissivity (T') from the aquifer to the river by use of a transmissivity factor (F_T).



Range of Aquifer Hydraulic and Geometric Values Modelled Asuming No Abstraction and with Emphasis on Analysing Baseflow Yield to the River Cell

K = Hydraulic Conductivity from 1 to 50m/d

S = Storativity from 0.01 to 0.25

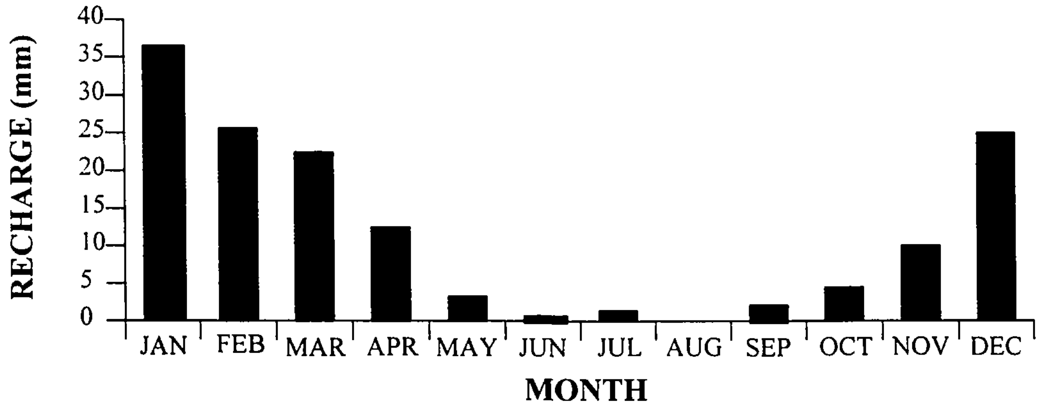
h_0= Fixed Head at River Cell from 1 to 50m

L = Aquifer Length from 300 to 5000m

<u>Other Variables Briefly Examined Include:</u>

Abstraction:	Quantity, Location and Season
Variable K:	Spatially Variable Hydraulic Conductivity
Variable K_b:	Variable River Bed HydraulicConductivityProperties Leading to Contrasting Conductance Terms

Figure 2.1 Simple Model (MODFLOW) Configuration



The Above 'Recharge' Distribution Represent the Average Monthly Effective Rainfall Calculated for MORECS Square 109 over the Period 1961-1990

For Most 'Dynamic' Model Runs the Above Monthly Average Distribution has been used as Input 'Recharge' for the Quasi-Steady-State Analysis

For Some Dynamic Model Runs the Historic Monthly Distribution has been used as Input 'Recharge' for Dynamic Historical Analysis

Figure 2.2 Average Monthly 'Recharge' from MORECS' Square 109

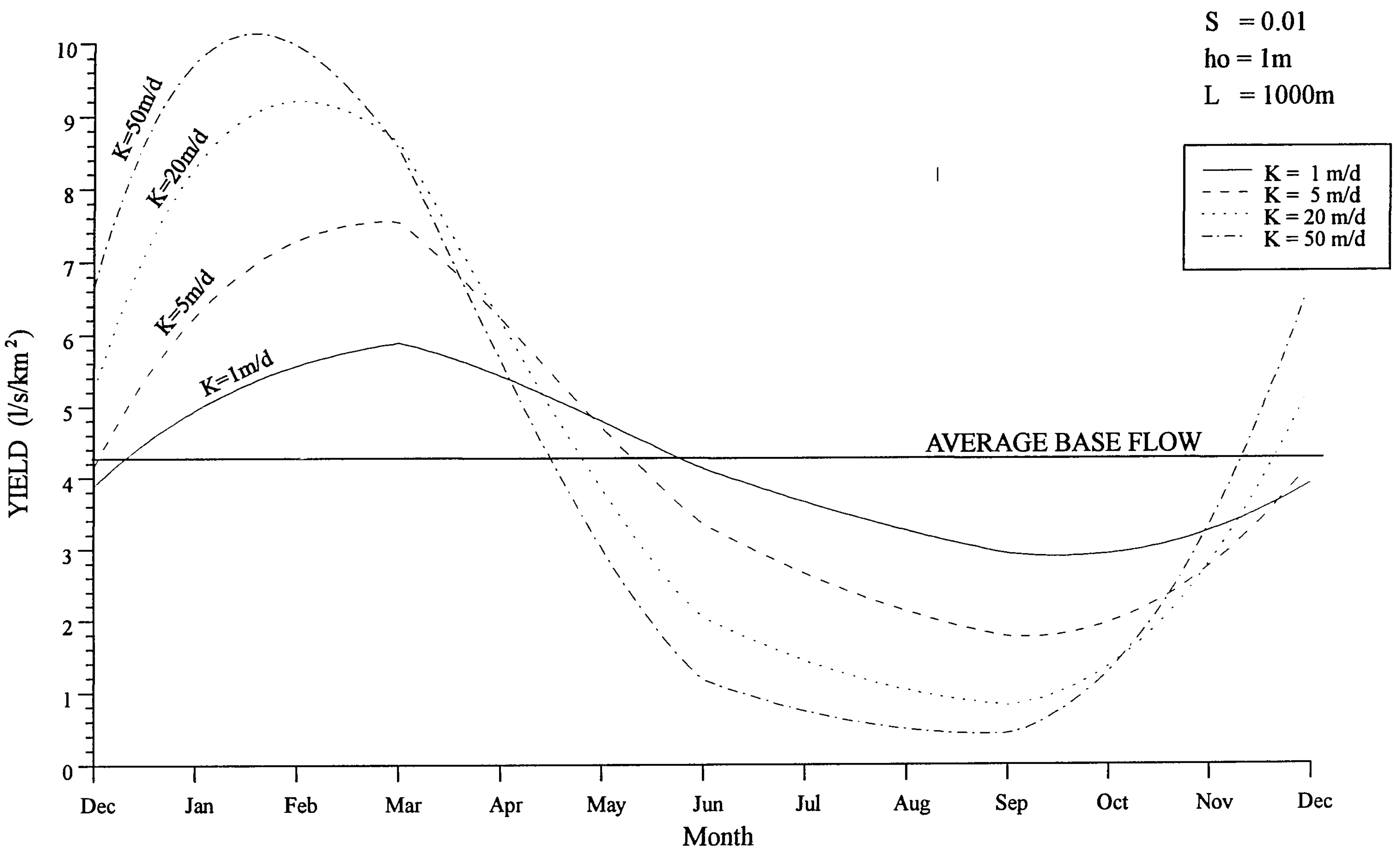


Figure 2.3 Quasi-Steady State Baseflow Yield (Year 10 - Low Storage)

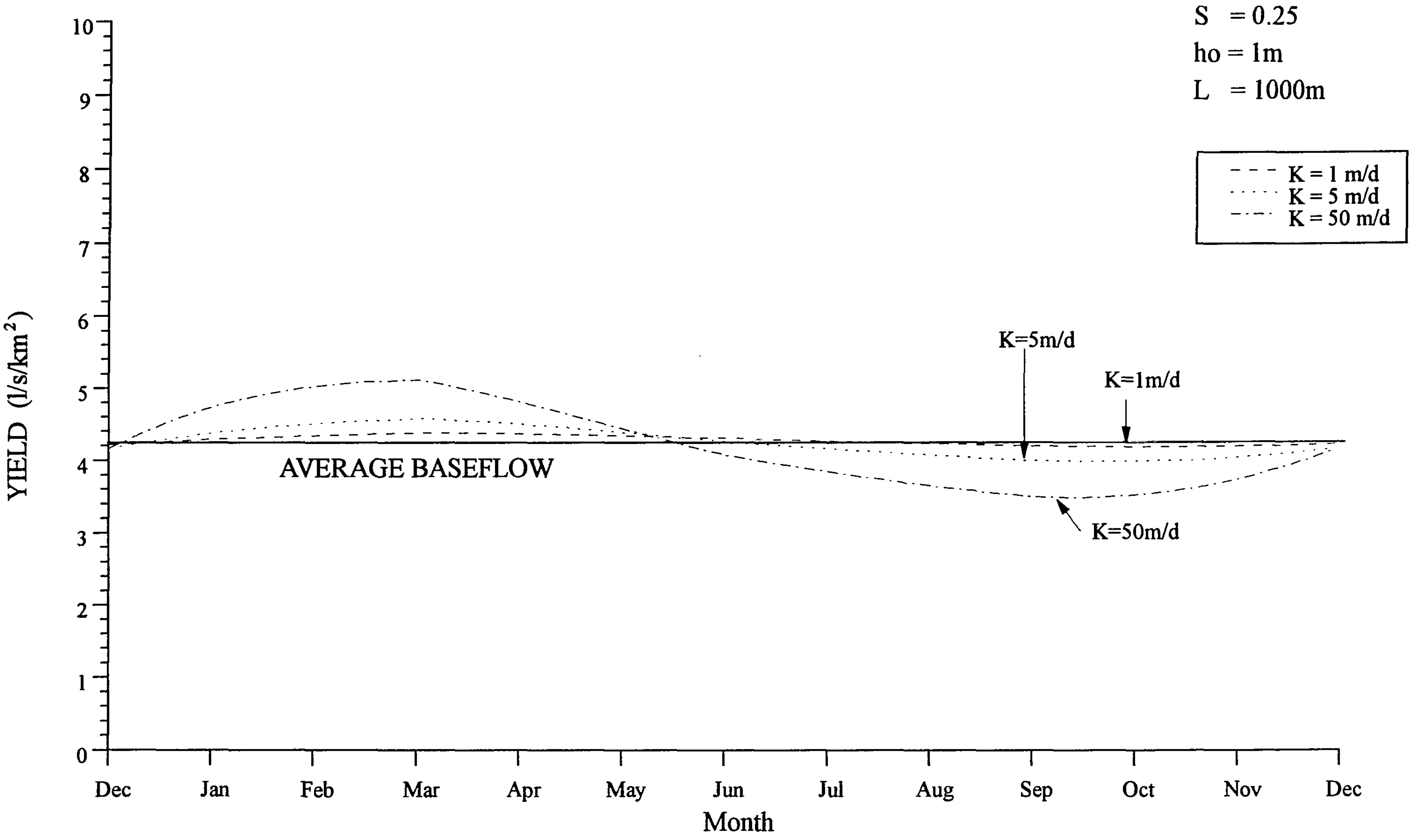


Figure 2.4 Quasi-Steady-State Baseflow Yield (Year 10 - High Storage)

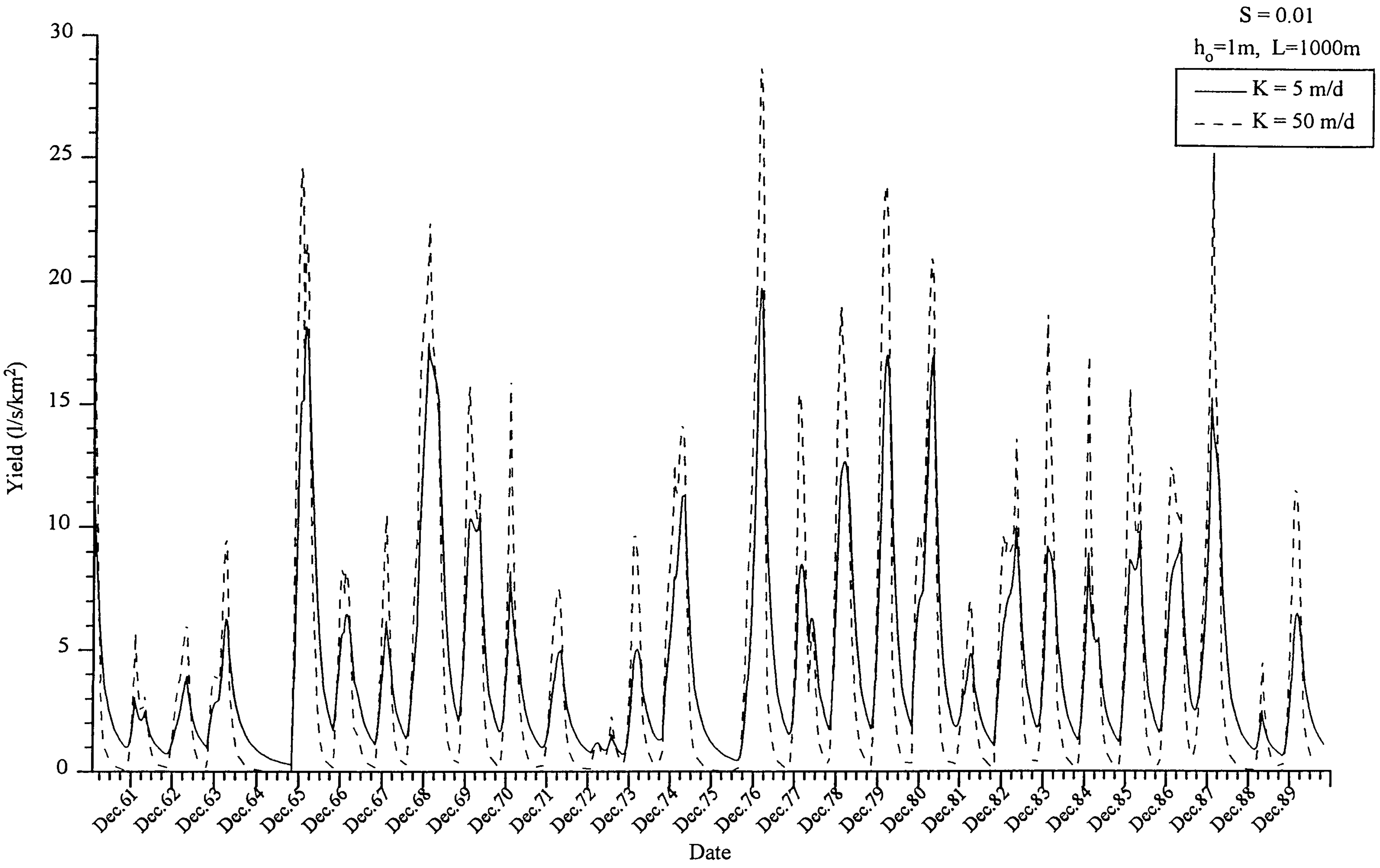


Figure 2.5 Dynamic Historic Baseflow Yields (Low Storage)

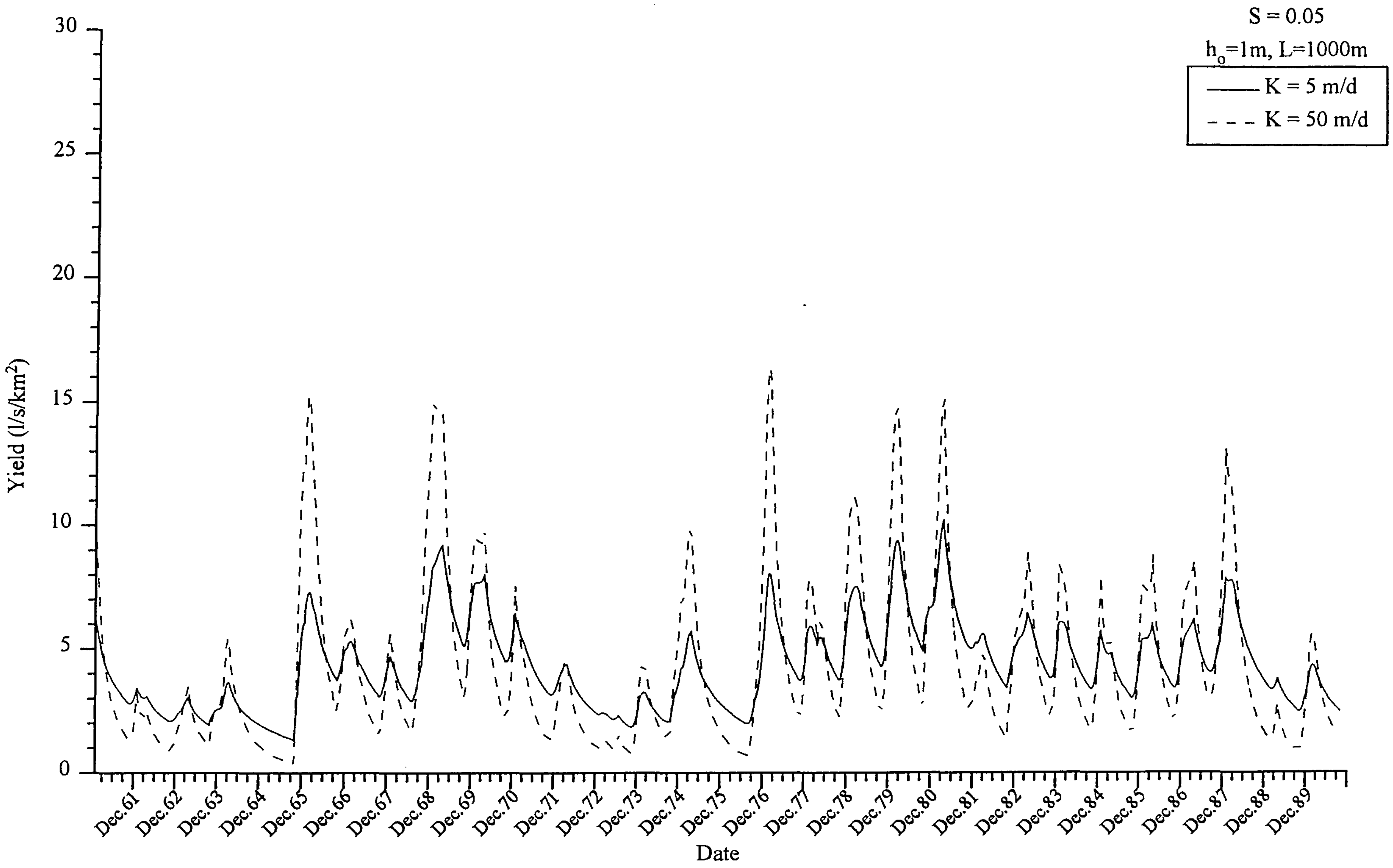


Figure 2.6 Dynamic Historic Baseflow Yields (Medium Storage)

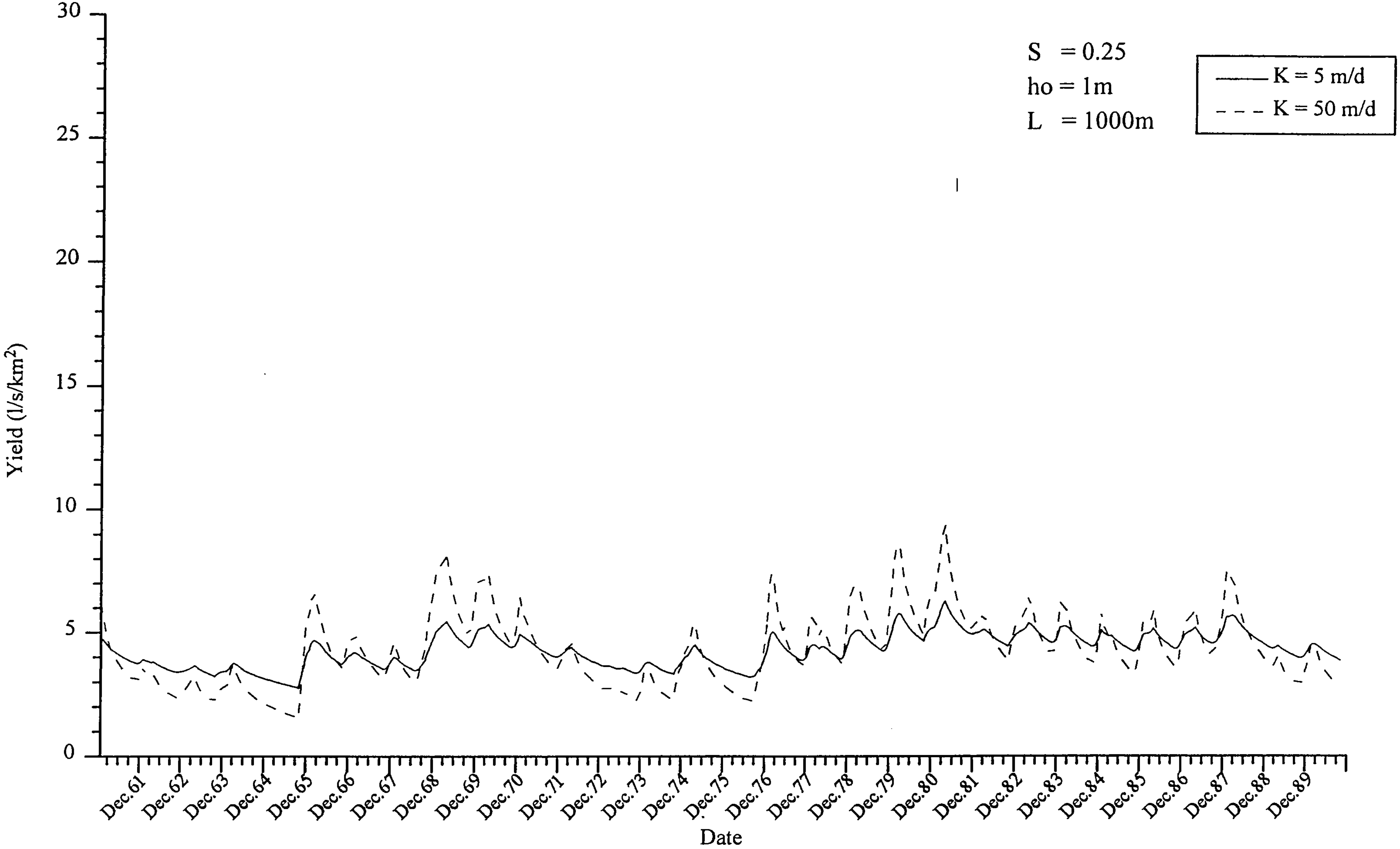
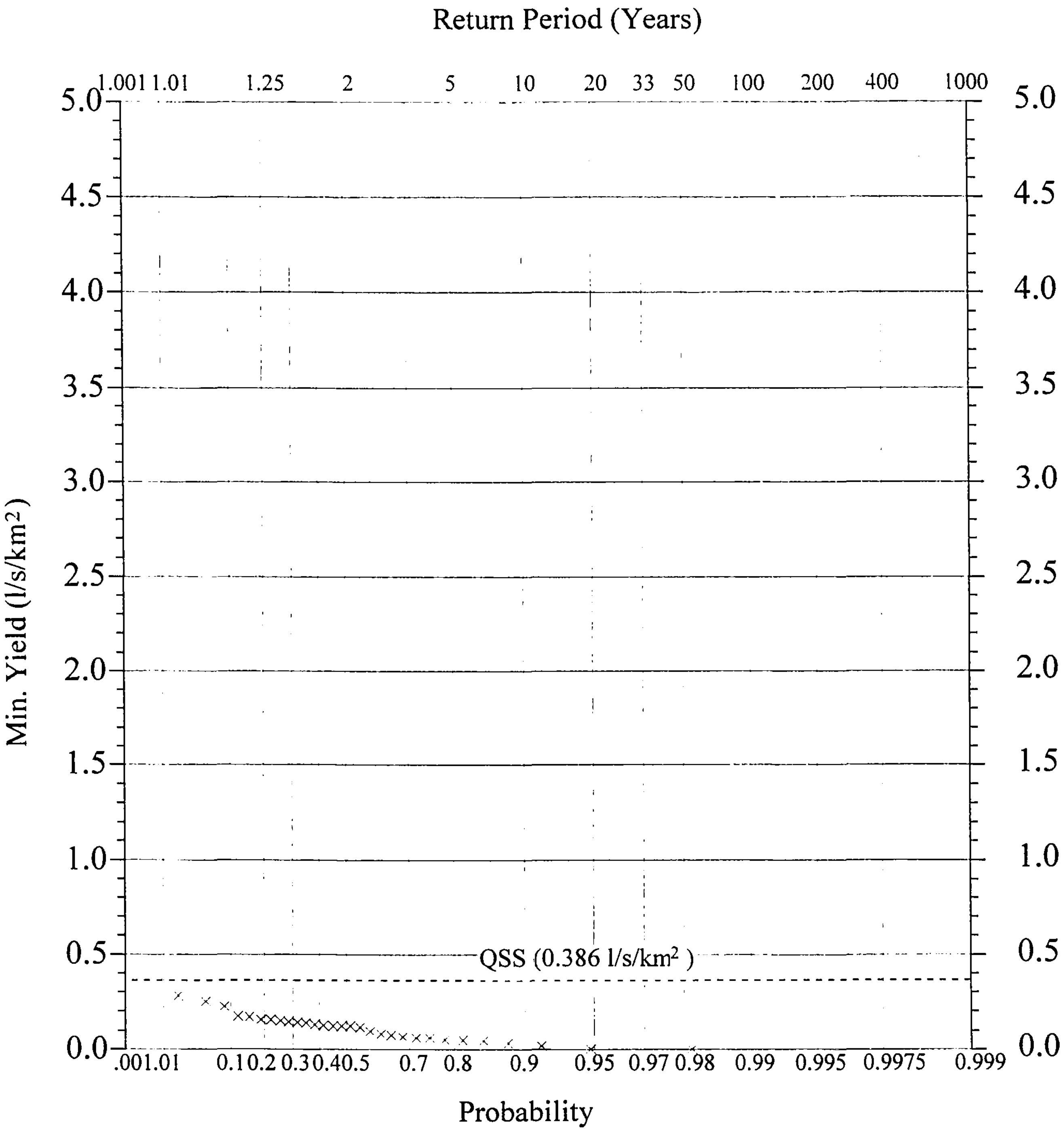


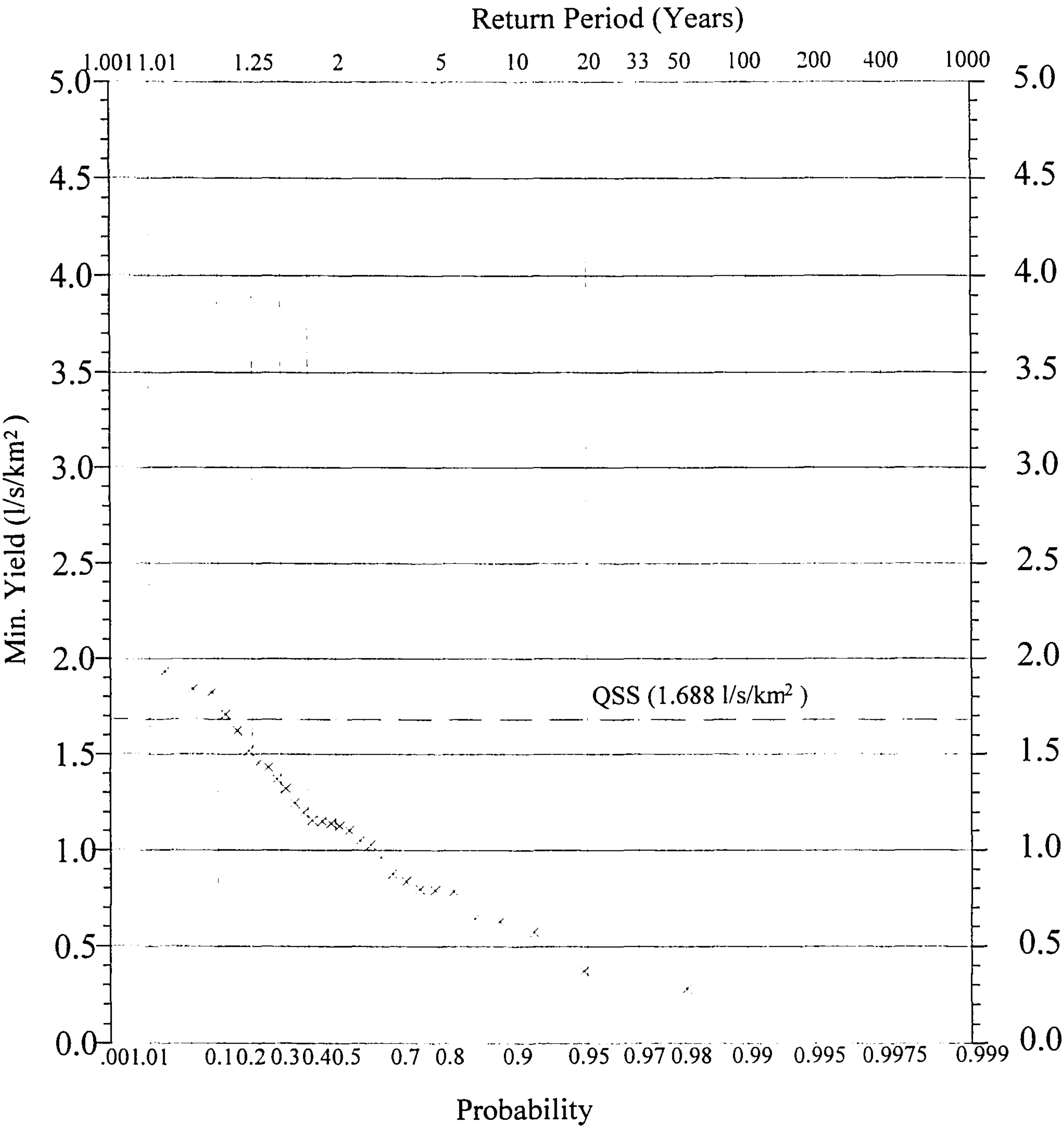
Figure 2.7 Dynamic Historic Baseflow Yields (High Storage)



QSS = Quasi-Steady-State Solution

ho = 1m
L = 1000m

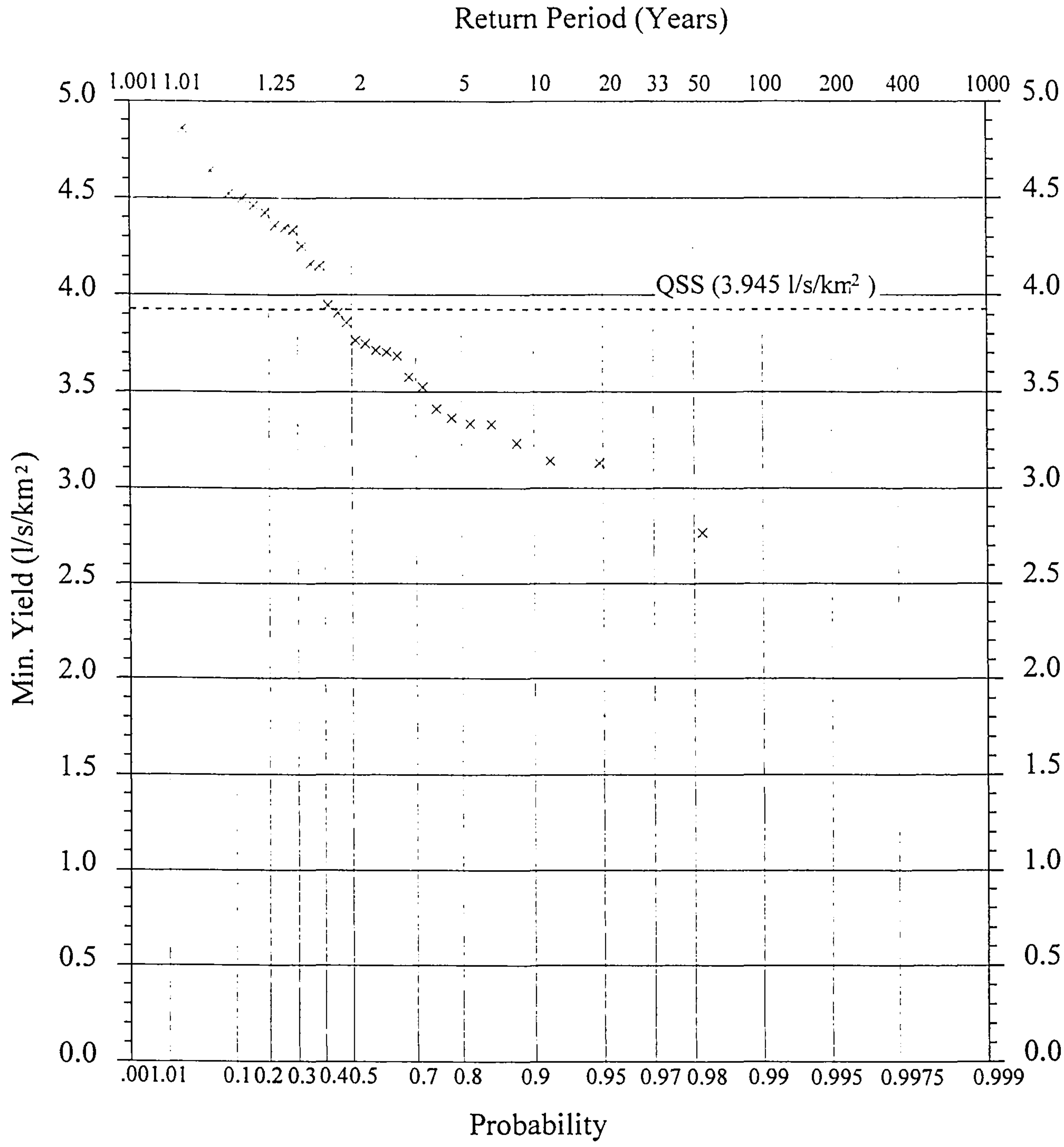
Figure 2.8 Probability Distribution of Minimum Yield Dynamic Historic Analysis (S=0.01, K=50m/d)



QSS = Quasi-Steady-State Solution

ho = 1m
L = 1000m

Figure 2.9 Probability Distribution of Minimum Yield Dynamic Historic Analysis (S=0.01, K=5m/d)



QSS = Quasi-Steady-State Solution

ho = 1m

L = 1000m

Figure 2.10 Probability Distribution of Minimum Yield Dynamic Historic Analysis (S=0.25, K=5m/d)

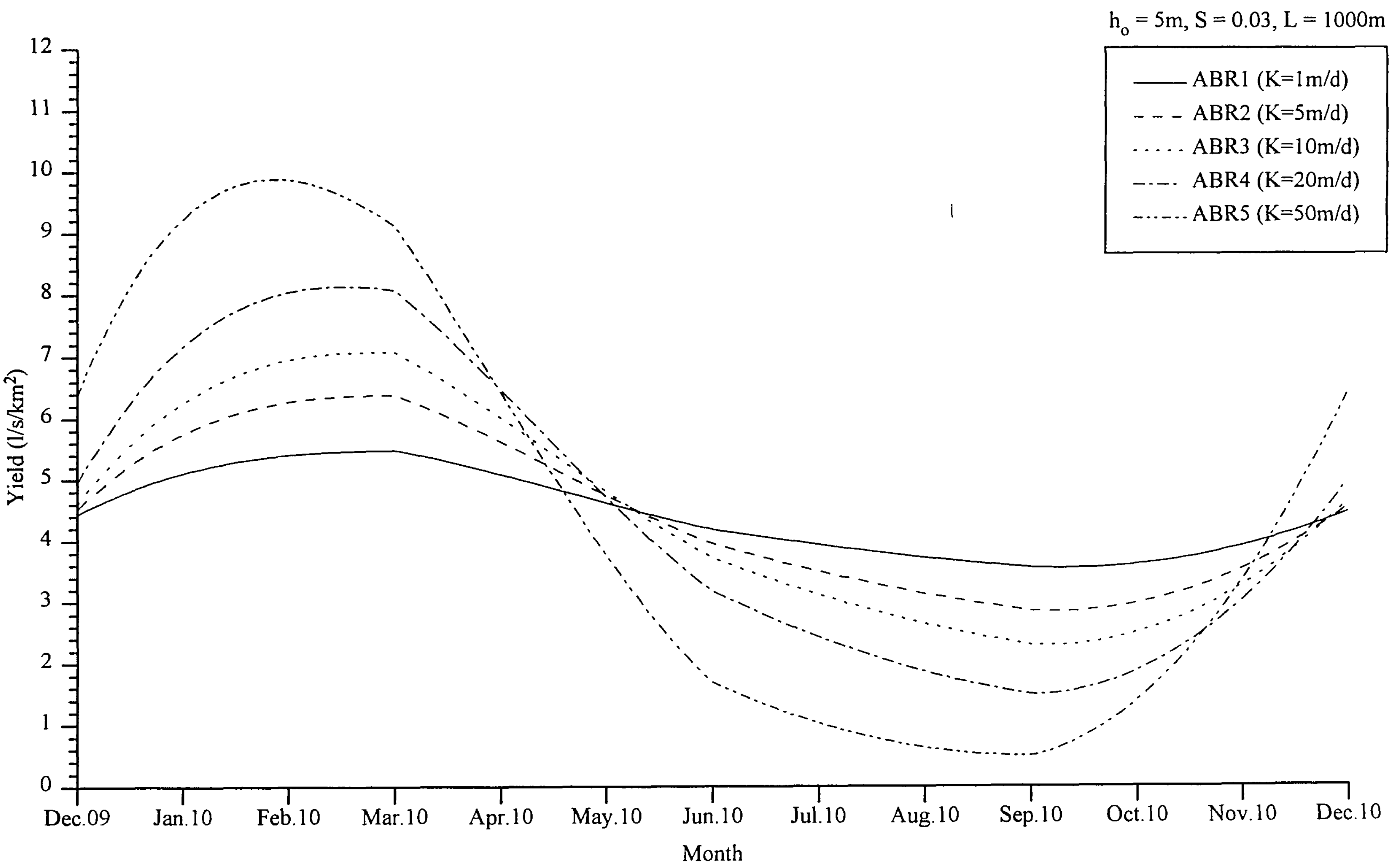


Figure 2.11 Quasi-Steady-State Additional Baseline Runs

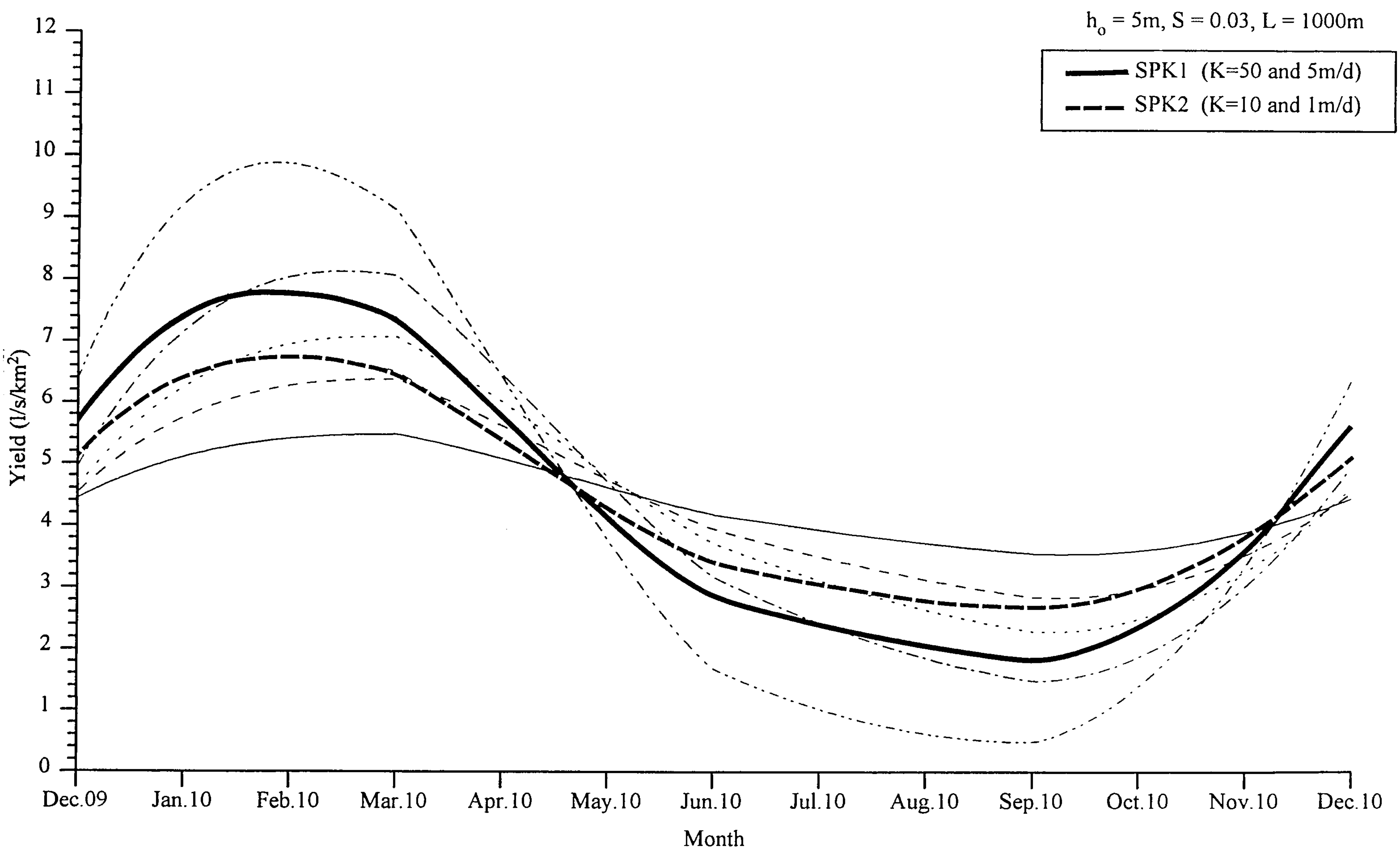


Figure 2.12 Spatially Variable Hydraulic Conductivity Runs Compared with Baseline Output (see Figure 2.11)

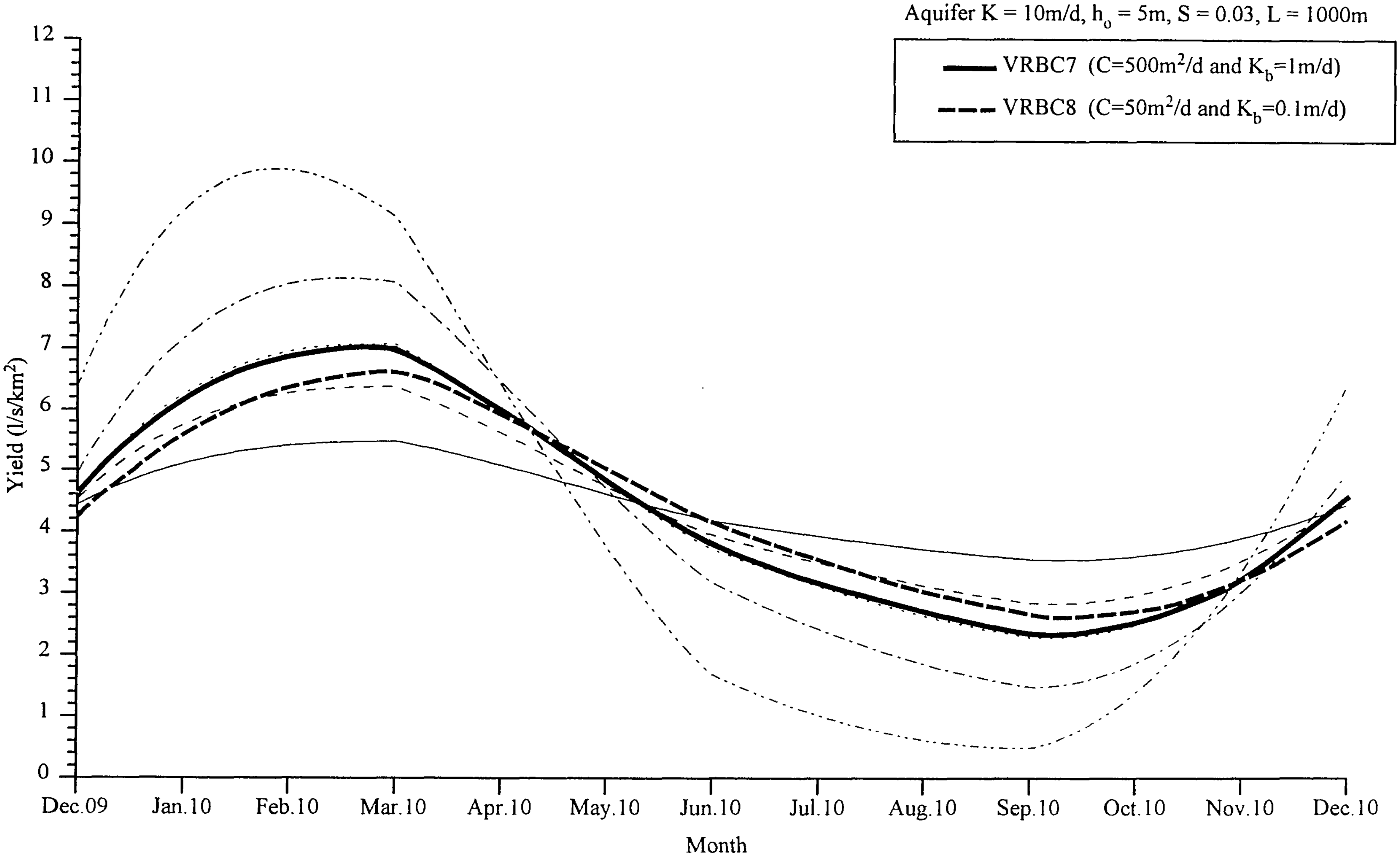


Figure 2.13 Variable River Bed Conductance (Aquifer K = 10 m/d) Compared with Baseline Output (see Figure 2.11)

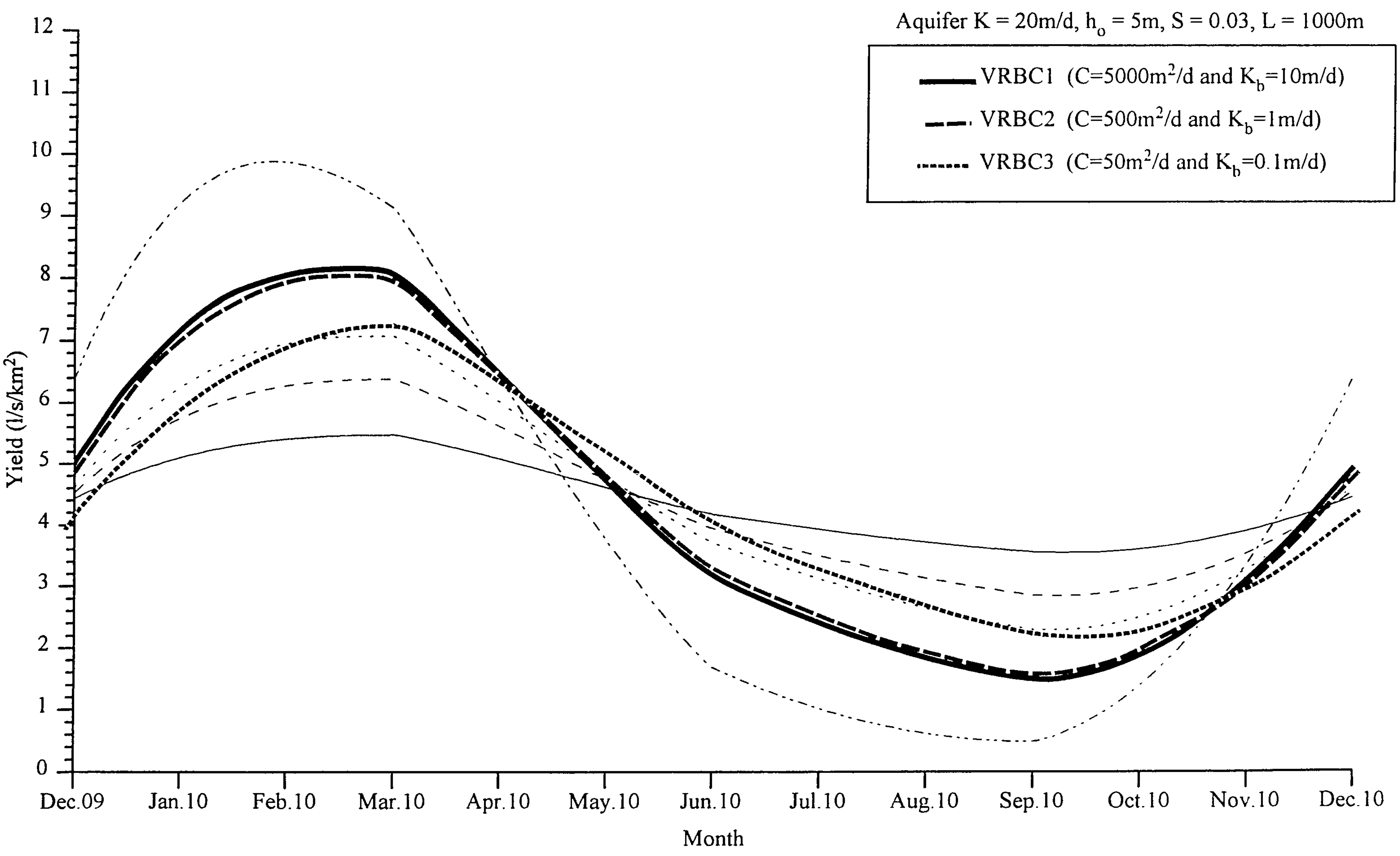


Figure 2.14 Variable River Bed Conductance (Aquifer K = 20 m/d) Compared with Baseline Output (see Figure 2.11)

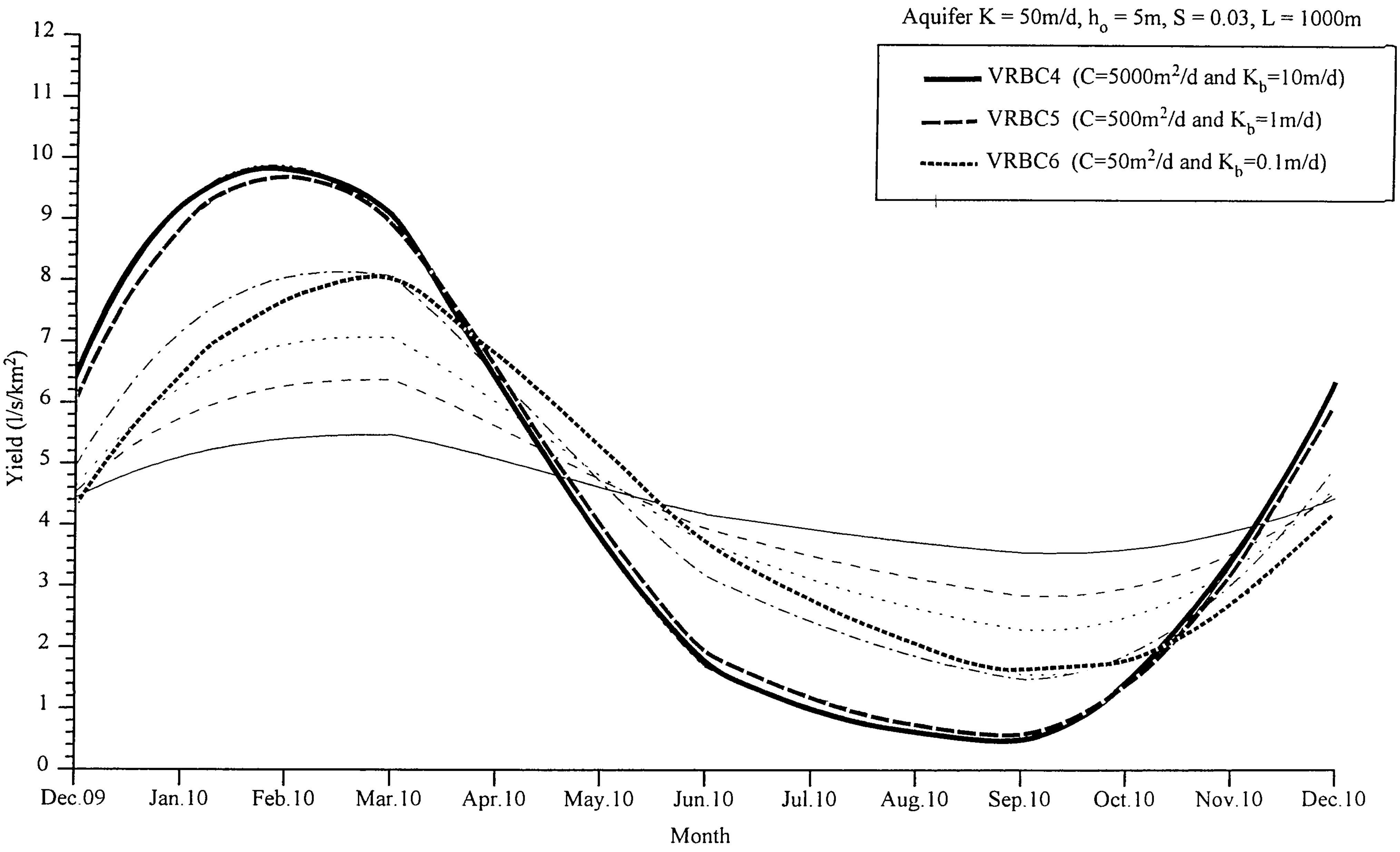


Figure 2.15 Variable River Bed Conductance (Aquifer K = 50 m/d) Compared with Baseline Output (see Figure 2.11)

3. ANALYTICAL CONSIDERATIONS

Consideration of the mathematical solution to the idealised aquifer problem (see Figure 2.1) has produced a formula which compares reasonably well with modelled minimum baseflows. The analytical problem considered was that of a constant transmissivity aquifer undergoing sinusoidal recharge at all points.

The resulting analytical solution is not fully verified though the crucial parameter T/L^2S, referred to as 'alpha' is shown to have direct parallels with much earlier work (Oakes and Wilkinson, 1972) which presents a similar parameter 'omega' (ω) where:

$$\omega = \frac{T\pi^2}{4SL^2}$$

In this earlier work recharge was simplified to a seasonal winter 'block' as opposed to the sinusoidal approximation adopted in this project.

The main breakthrough in this project, compared with earlier work, was to recognise how the 'alpha' term could be used to develop the type curve which is described further below.

Transferring the 'alpha' parameter to the unconfined aquifer situation is not strictly valid because T is not constant in the unconfined configuration. Experimentation with the known model results, for runs in which aquifer K is constant throughout the model and no river bed conductance is applied, established that the best approximator for T in the definition of alpha is Kh where $h^2 = ho^2 + qL^2/K$ (based on the solution to the steady-state Dupuit equation at $x = L$).

As shown in Figure 3.1 all the model runs for the various, K, ho, S and L values used fall roughly on the same line which is close but not perfectly the same as the analytical constant transmissivity aquifer solution. The reasons for the scatter are thought to be a combination of:

- Error in analytical solution assuming constant transmissivity
- Differences in analytical solution using sinusoidal (and not quasi-steady-state) recharge
- Error in modelled solution arising from numerical approximation.

The use of this graph as a type curve clearly provides a method of arriving at a reasonable estimate of D_T (minimum baseflow yield over average baseflow yield) given K, ho, L and S without having to run the model.

The alpha value of an aquifer is an indication of how well the aquifer acts as a reservoir in storing groundwater and regulating baseflow. In turn, this can be used to indicate the aquifer's abstraction development potential and this concept is expanded in Section 4 below. High alphas indicate rapid response low-flexibility aquifers and low alphas indicate flexible aquifers with respect to groundwater resource development potential.

Using the data compiled in Table 2.2 an attempt has been made to show that minimum baseflow yields and D_T values derived under real historic recharge conditions when compared with quasi steady state derivations are:

- slightly reduced for the median value
- further reduced for increasing return period.

This is shown in Figure 3.2 which highlights reductions for 20 year return periods and the median (mean annual) value.

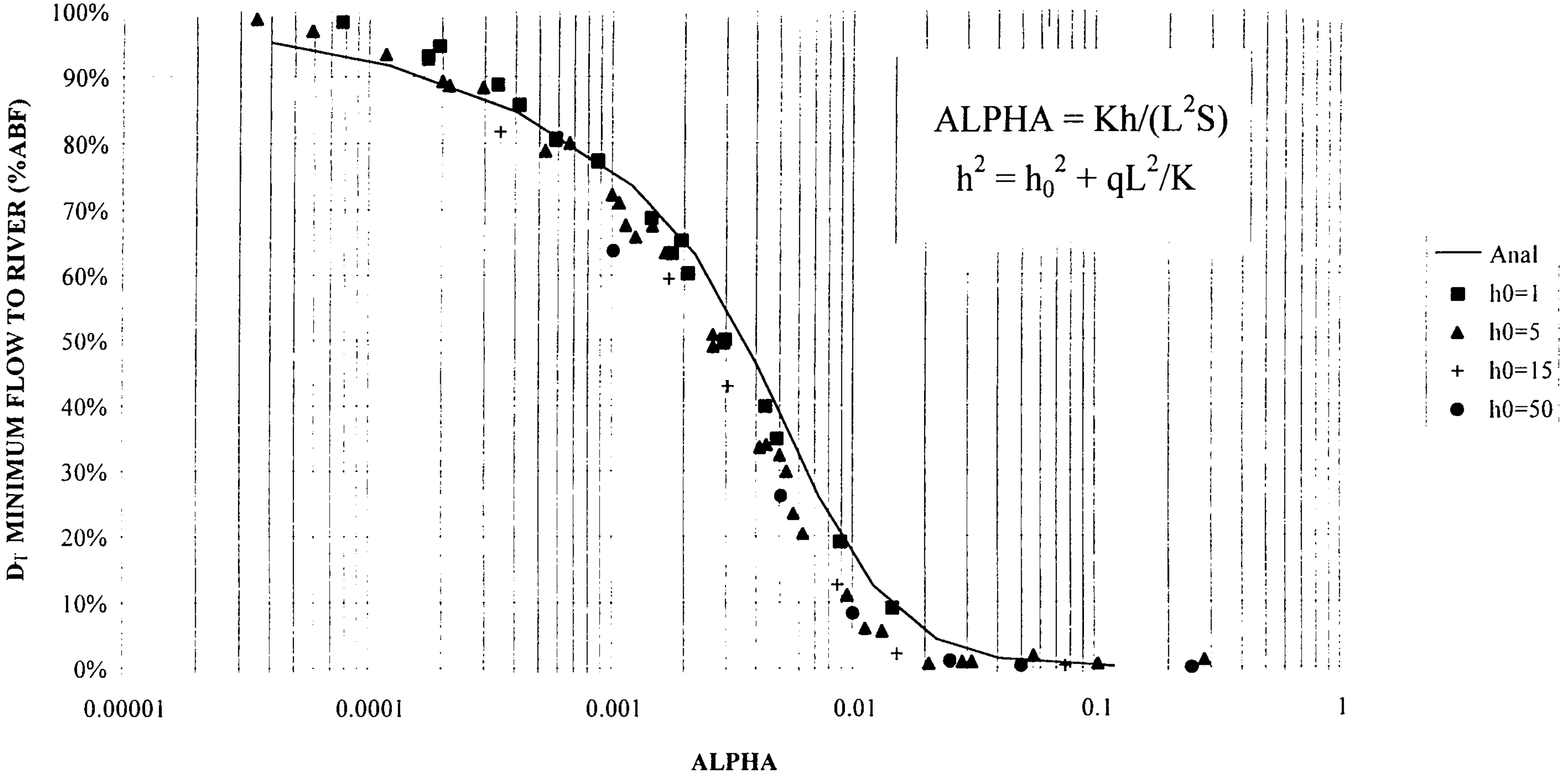
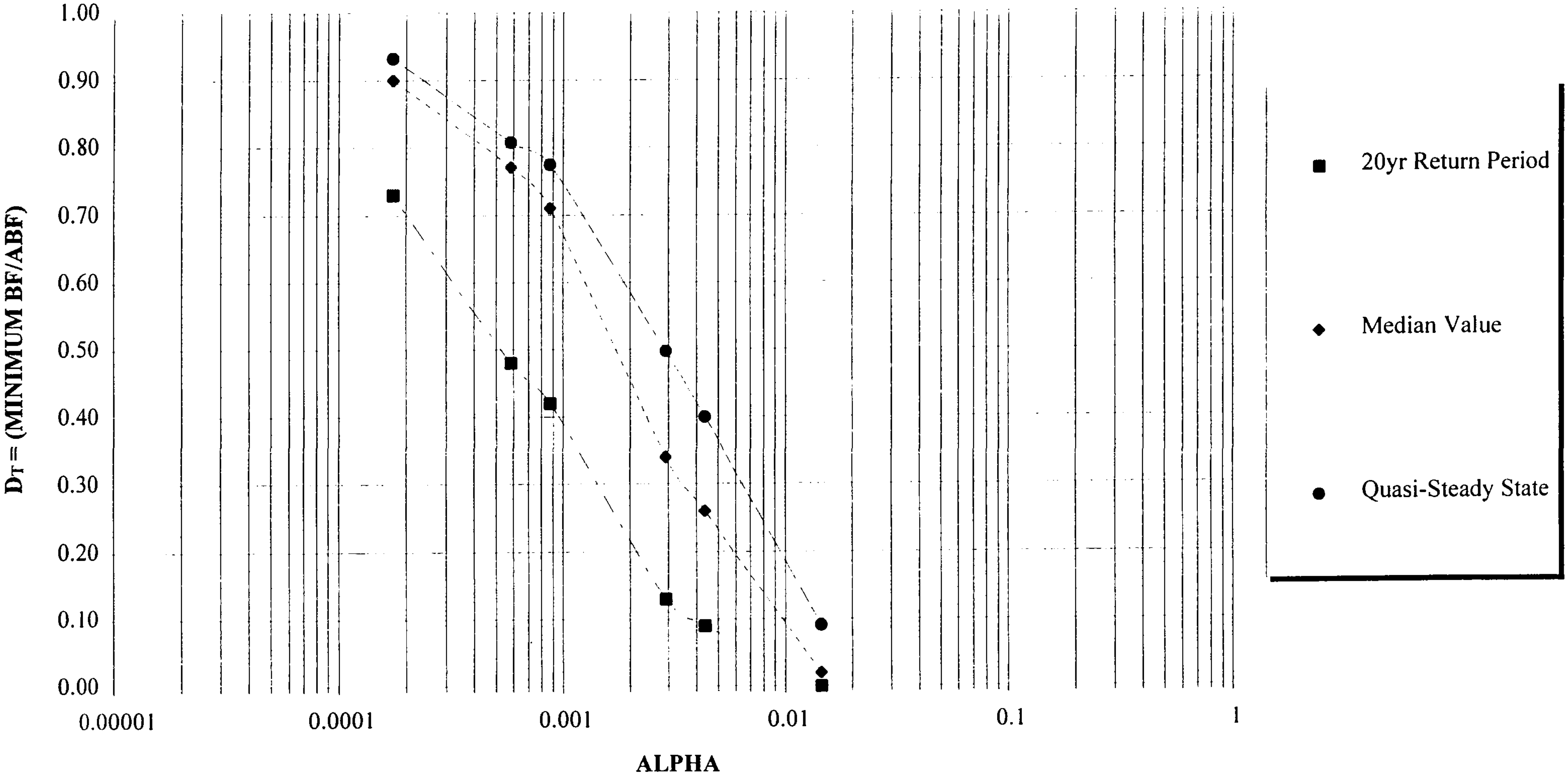


Figure 3.1 Comparison of Analytic/Modelled Solutions - Quasi-Steady-State Model



$$\alpha = Kh/(L^2S), \text{ where } h^2 = h_0^2 + qL^2/K$$

Figure 3.2 Comparison of 1 in 20 Year, Median and Quasi-Steady-State D_T Values

4. VERIFICATION OF D_T THEORY

Using data readily accessible in the Institute of Hydrology Yearbooks, an attempt has been made to verify the theories postulated for the relationship between D_T and the aquifer parameters used to define the 'alpha' term. A number of catchments have been selected on the basis that:

- they have rivers which are baseflow dominated
- believed not to experience overwhelming artificial influences
- some are well known to members of the project team.

For each catchment the values of the parameters have been selected as follows:

- Average Baseflow : BFI x Average Flow

- Minimum Flow : The minimum flows recorded from 1981 to 1990 have been averaged.

- Equivalent Aquifer Length: The aquifer length (L) used in modelling and analytical work previously described is replaced by the equivalent aquifer length (EAL). This is calculated by dividing half the Catchment Area by the total river length in the catchment. This is a recommended formula which should work well for simple geometrical aquifer and river configurations.

- 'Effective' Transmissivity: The 'effective' transmissivity (T') used is the product of aquifer transmissivity (T) (or Kh) and the transmissivity factor (F_T) which is derived from a brief assessment of (or assumptions about) river bed properties. The 'effective' transmissivity (T') is an approximation for 'transmissivity' between the aquifer and the river and has been found from modelling analysis to be an important controlling factor on D_T.

The results are given in Table 4.1 and Figure 4.1 shows the correlation between the calculated values of D_T and the theoretical values. The trend in the data is clearly visible although there is a fair amount of scatter. Some of the scatter may be explained by artificial influences on the river which are not taken into account. It is expected that with more accurate estimates of T', S and other parameters the scatter may be reduced. Further and more detailed case examples; which go on to examine *yield* are described in Section 3.2 of the User Manual (Papaioannou and Erskine, 1996a).

Table 4.1 Comparison of observed and theoretical reduction factors

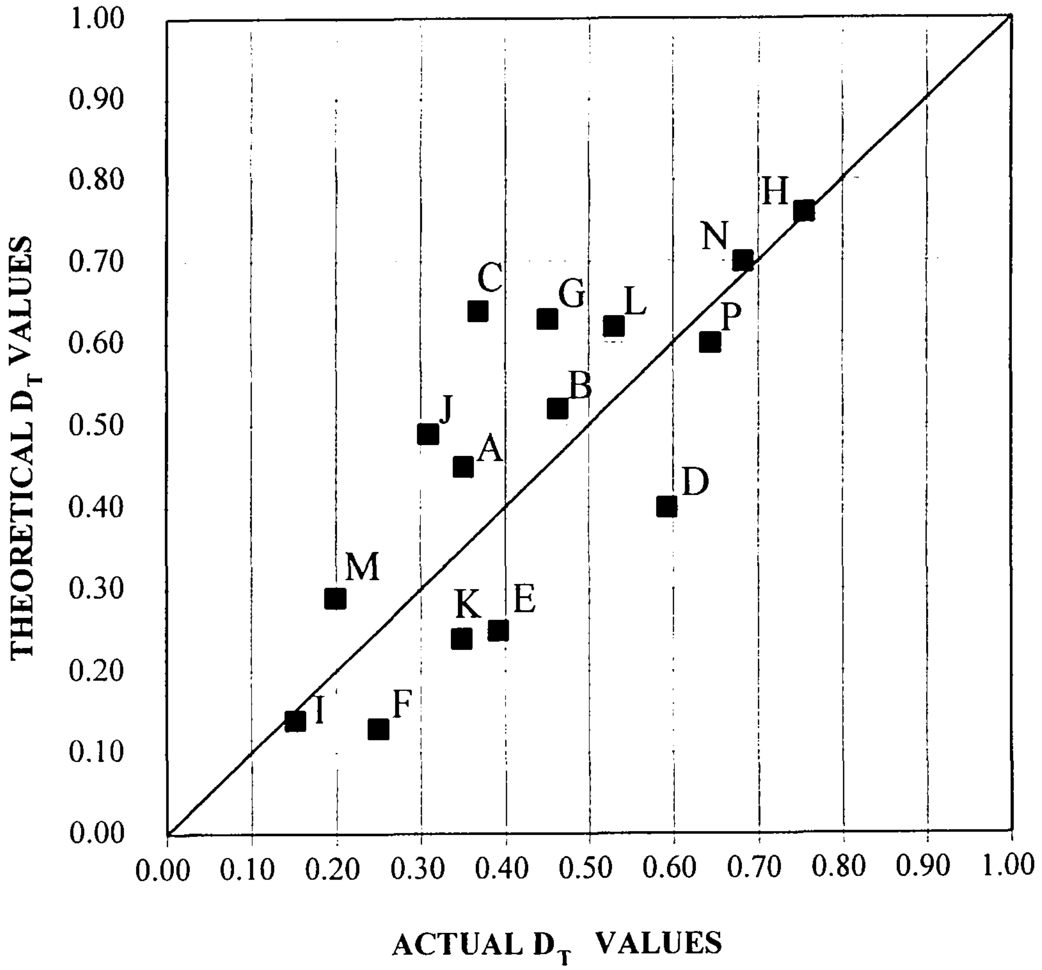
ACTUAL GAUGED DATA
From Hydrological Data Books 1981-85 and 1986-90

THEORETICAL VALUES
From aquifer parameters and Type Curve

Chart Ref.	Station Number	River	Area	BFI	ADF	ABF	Minimum Flow 1986 1981	1987 1982	1988 1983	1989 1984	1990 1985	Avg = MBF	Rf =MBF/ABF	Aquifer	T	T'	S	River Length	EAL	Alpha $\alpha = T'/S*EAL^2$	Rf
			km^2		m^3/s	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s	m^3/s			m^2/d	m^2/d	-	km	km	/d	
A	26003	Foston Beck	57.2	0.96	0.68	0.65	0.29	0.29	0.23	0.09	0.06	0.228	0.35	Chalk	800	800	0.04	13	2.20	0.00413	0.45
							0.3	0.2	0.25	0.28	0.29										
B	27056	Pickering Beck	68.6	0.69	0.86	0.59	0.34	0.35	0.32	0.17	0.15	0.274	0.46	Lst/Grit	100	100	0.08	55.5	0.62	0.00327	0.52
							0.38	0.25	0.26	0.23	0.29										
C	28015	Idle	529	0.79	3.89	3.07	1.28	1.8	1.32	0.75	0.49	1.128	0.37	S.Sst	1000	900	0.15	159	1.66	0.00217	0.64
D	29002	Great Eau	77.4	0.88	0.68	0.60		0.46	0.40	0.28	0.24	0.354	0.59	Chalk	600	540	0.04	23.4	1.65	0.00494	0.40
							0.40	0.35	0.36	0.33	0.37										
E	29003	Lud	55.2	0.9	0.48	0.43	0.17	0.27	0.2	0.1	0.14	0.169	0.39	Chalk	600	600	0.04	19.5	1.42	0.00749	0.25
							0.22	0.16	0.16	0.12	0.15										
F	30013	Heighington Beck	21.2	0.75	0.15	0.11	0.03	0.05	0.02	0.02	0.02	0.028	0.25	Lincs Lst	600	600	0.04	10	1.06	0.01335	0.13
							0.03	0.03	0.02	0.03	0.03										
G	39028	Dun	101.3	0.95	0.76	0.72	0.36	0.32	0.36	0.24	0.22	0.325	0.45	Chalk	600	600	0.04	19	2.67	0.00211	0.63
							0.42	0.3	0.38	0.29	0.36										
H	39029	Tillingbourne	59	0.89	0.56	0.50	0.39	0.37	0.45	0.34	0.29	0.376	0.75	Lwr Grsnd	300	300	0.15	20	1.48	0.00092	0.76
							0.4	0.36	0.43	0.35	0.38										
I	39042	Leach	76.9	0.78	0.77	0.60	0.07	0.06	0.11	0.02	0.01	0.091	0.15	Ool Lst	800	800	0.04	30	1.28	0.01218	0.14
							0.15	0.1	0.1	0.04	0.25										
J	39043	Kennett	295	0.95	2.59	2.46	0.9	0.85	0.79	0.42	0.33	0.757	0.31	Chalk	800	800	0.04	62	2.38	0.00353	0.49
							1.3	0.72	0.75	0.59	0.92										
K	39076	Windrush	296	0.84	2.29	1.92	0.8	0.65	0.74	0.4	0.42	0.668	0.35	Ool Lst	800	660	0.04	103	1.44	0.00799	0.24
							0.93	0.59		0.51	0.97										
L	42004	Test	1040	0.95	11.5	10.93	6.26	6.15	6.89	4.95	5.08	5.785	0.53	Chalk	800	720	0.04	136	3.82	0.00123	0.62
							5.94	6.08	5.9	4.84	5.76										
M	53028	Ry Brook	102	0.75	1.56	1.17	0.26	0.21	0.37	0.18	0.14	0.232	0.20	Ool Lst	400	380	0.04	61	1.20	0.00660	0.29
N	54044	Tern	92.6	0.76	0.87	0.66	0.44	0.54	0.52	0.37	0.36	0.451	0.68	S.Sst	600	540	0.15	30	1.54	0.00151	0.70
							0.5	0.46	0.48	0.38	0.46										
P	(45005 - 45008)	Otter *	98.3	0.59	1.09	0.64	0.39	0.39	0.43	0.33	0.34	0.414	0.64	S.Sst	150	120	0.15	82.5	0.60	0.00225	0.60
							0.46	0.55	0.42	0.36	0.47										

BFI = Base Flow Index
ADF= Average Daily Flow (as gauged)
ABF= Average Baseflow
MBF= Minimum Baseflow
D_T = MBF/ABF
* Composite Record

T = Aquifer Transmissivity
T'= "Effective" Transmissivity
S = Storage
EAL = Equivalent Aquifer Length = 0.5*Area/River Length
D_T = From Alpha/D_T graph (Figure 3.1)



Data Labels refer to Table 4.1

Figure 4.1 Comparison of Actual and Theoretical Minimum Baseflow/Average Baseflow (D_T) Values

5. REFERENCES

Papaioannou and Erskine, 1996a. Environment Agency R&D Project 544 Technical Report W9 - User Manual. Groundwater Resource Reliable Yield.

Papaioannou and Erskine, 1996b. Environment Agency R&D Project 544 Technical Report W10 - Key Components in Methodology Formulation. Groundwater Resource Reliable Yield.

D B Oakes and W B Wilkinson, Water Resources Board (WRB), 1972. Modelling of Groundwater and Surface Water Systems.